Business cycle and sector cycles

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Abstract

A methodology based on the multivariate generalized Butterwoth filter for extracting the business cycles of the whole economy and of its productive sectors is developed. The method is then illustrated through an application to the Italian gross value added time series of the main economic sectors.

1 Introduction

The definition of business cycle generally accepted in economic literature is "fluctuations of period ranging from 1.5 to 8 years, involving different sectors and aspects of the economic activity". This implicitly means that movements of period longer than 8 years are to attribute to structural reasons rather then to conjunctural ones.

In modern countries there are automatic stabilizers, such as progressive taxation, unemployment benefits, etc., and discretionary counter-cyclical policies. Thus, it is fundamental that policy makers have tools to distinguish structural problems from cyclical contractions, since counter cyclical policies should have only short run effects. Long run changes can be achieved only through structural reforms.

Efficient economic policies are those which accomplish a given result with the minimum cost. Observing the quarterly time series of the value added (VA) of different sectors, it is clear that the trends vary considerably from sector to sector. Some sectors grow and some decline. At the same time, even though not so obvious from the visual inspection of the time series, there are sectors strongly dependent on the macro-business cycle and sectors for which the idiosyncratic cycle takes account of the greatest part of variability at business cycle frequencies. For these reasons counter-cyclical policies may be considered efficient only if they affect sectors which in a given moment of time are in a contraction phase.

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The present work is an original application of recent developments in structural time series modeling (Harvey and Trimbur 2003) to the analysis of the trends, the macro-business cycle and sector-business cycles of a pool of sector time series. The developed methodology is then applied to the main sectors of the Italian economy.

2 Structural models and Butterworth filters

The approach here undertaken is the one recently proposed by Harvey and Trimbur (2003), which, in the construction of linear filters for the extraction of trend and cycle, pursues two contemporaneous goals:

- ▷ building linear filters that are good approximations to ideal filters[1],
- ▷ building linear filters corresponding to stochastic components meaningful from the point of view of econometric theory and usable for forecasting.

The two goals are generally in contrast with each other, since linear filters that well approximate ideal filters imply stochastic components with a high order of integration, while econometricians are unanimous in considering not realistic a trend with integration order greater then 2 or an integrated business cycle.

2.1 Butterworth filters

The low-pass Butterworth filter is defined, in time domain, by

$$B_m^{\mathrm{lp}}(L) = \frac{1}{q^{-1}[(1-L)(1-L^{-1})]^m}, \tag{1}$$

with L, lag operator, and $m \in \mathbb{N}$. The gain of the filter is

$$B_m^{\mathrm{lp}}(e^{-i\lambda}) = \frac{1}{q^{-1}(2-2\cos\lambda)^m}. \tag{2}$$

Setting $q = [2\sin(\lambda_{\mathrm{lp}}/2)]^{2m}$ with $0 < \lambda_{\mathrm{lp}} < \pi$ and using trigonometric identities, one gets the usual form of the Butterworth filter:

$$B_m^{\mathrm{lp}}(e^{-i\lambda}) = \left[1 + \left(\frac{\sin(\lambda/2)}{\sin(\lambda_{\mathrm{lp}}/2)}\right)^{2m}\right]^{-1}. \tag{3}$$

As shown in figure 1, the parameter λ_{lp} determines the point in which the gain is $1/2$, while m controls the slope of the gain in λ_{lp}: the higher m, the more rectangular is the low-pass filter's gain and the more accurate is the approximation to the ideal filter.

A standard transform from digital signal analysis (Pollock 1999, example 16.5) can be used to build a band-pass filter from a low-pass filter. The application of

[1] A filter is ideal if its gain function is an indicator function over the selected band.

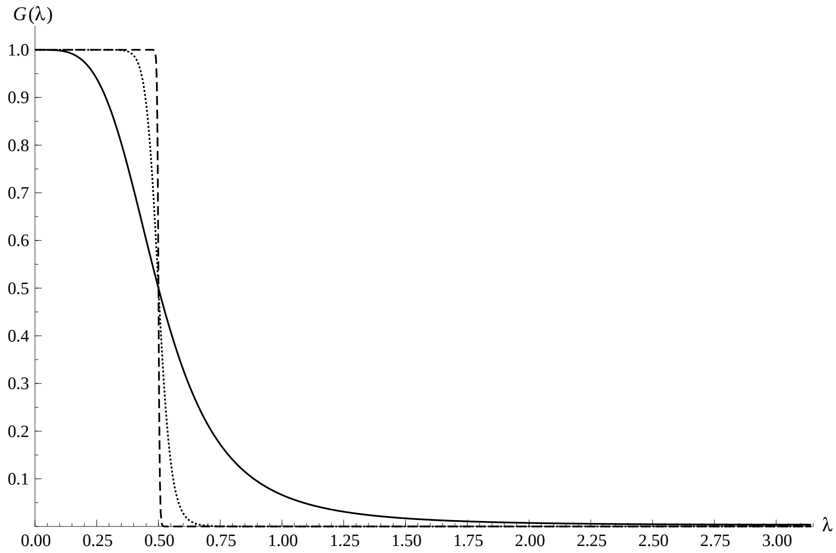

Figure 1: Gain of the Butterworth low-pass filter for $\lambda_{\mathrm{lp}} = 0.5$ and $m = 2$ (solid), $m = 10$ (dotted) e $m = 100$ (dashed).

this transformation to the Butterworth band-pass filter produces a new filter with gain function

$$B^{bp}_{n,\lambda_c}(e^{-i\lambda}) = \left[1 + \frac{1}{q}\left(\frac{4(\cos\lambda - \cos\lambda_c)^2}{1 + \cos^2\lambda_c - 2\cos\lambda_c\cos\lambda}\right)^n\right]^{-1}, \tag{4}$$

where $\lambda_c \in (0, \pi)$ is the modal frequency, $q \in [0, \infty)$ is a parameter that determines the band width and $n \in \mathbb{N}$ has been used instead of m, since m will be kept in the following for the order of the low-pass filter. Thus, the Butterworth band-pass filter has three parameters that make it rather flexible: λ_c is a location parameter, q determines the band width (figure 3) and n controls how rectangular is the gain (figure 2).

Consider now the process

$$y_t = \varsigma_t + \varepsilon_t,$$

where ς_t is the signal and ε_t is white noise with variance σ^2_ε. The Butterworth band-pass filter is MSE-optimal for extracting the signal, if ς_t is the process $\mu_{m,t}$ defined by

$$\begin{aligned} \mu_{1,t} &= \mu_{1,t-1} + \zeta_t, \\ \mu_{i,t} &= \mu_{i,t-1} + \mu_{i-1,t-1}, \quad i = 2, \ldots, m, \end{aligned} \tag{5}$$

with $\zeta_t \sim WN(0, \sigma^2_\zeta)$, and where in equation (2) $q = q_\zeta = \sigma^2_\zeta/\sigma^2_\varepsilon$. Following Harvey and Trimbur (2003), $\mu_{m,t}$ will be referred to as *m-order stochastic trend*.

For $m = 1$ $\mu_{m,t}$ is a random walk, while for $m = 2$ it is an integrated random walk. Both processes are often used for modeling trends in economic time series (Harvey 1989, West and Harrison 1989).

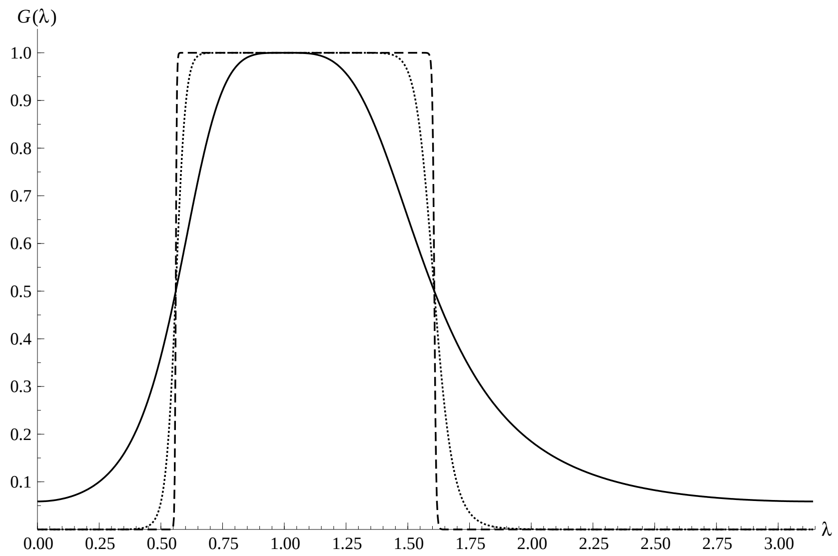

Figure 2: Gain of the Butterworth band-pass filter for $\lambda_c = 1$, $q = 1$ and $n = 2$ (solid), $n = 10$ (dotted) and $n = 100$ (dashed).

The Butterworth band-pass filter is MSE-optimal for extracting the signal, when ς_t is the process $\psi_{n,t}$ defined by

$$\begin{bmatrix} \psi_{1,t} \\ \psi^*_{1,t} \end{bmatrix} = \begin{bmatrix} \cos\lambda_c & \sin\lambda_c \\ -\sin\lambda_c & \cos\lambda_c \end{bmatrix} \begin{bmatrix} \psi_{1,t-1} \\ \psi^*_{1,t-1} \end{bmatrix} + \begin{bmatrix} \kappa_t \\ 0 \end{bmatrix},$$

$$\begin{bmatrix} \psi_{i,t} \\ \psi^*_{i,t} \end{bmatrix} = \begin{bmatrix} \cos\lambda_c & \sin\lambda_c \\ -\sin\lambda_c & \cos\lambda_c \end{bmatrix} \begin{bmatrix} \psi_{i,t-1} \\ \psi^*_{i,t-1} \end{bmatrix} + \begin{bmatrix} \psi_{i-1,t} \\ 0 \end{bmatrix}, \tag{6}$$

$$i = 2, \ldots, n,$$

with κ_t white noise with variance σ^2_κ and where in equation (4) $q = q_\kappa = \sigma^2_\kappa/\sigma^2_\varepsilon$. The problem with the now defined component ψ_n is that it is an integrated process, while the business cycle to be extracted through the band-pass filter is supposed to be stationary. Harvey and Trimbur (2003) propose a modification of equation (6) that allow stationarity:

$$\begin{bmatrix} \psi_{1,t} \\ \psi^*_{1,t} \end{bmatrix} = \rho \begin{bmatrix} \cos\lambda_c & \sin\lambda_c \\ -\sin\lambda_c & \cos\lambda_c \end{bmatrix} \begin{bmatrix} \psi_{1,t-1} \\ \psi^*_{1,t-1} \end{bmatrix} + \begin{bmatrix} \kappa_t \\ 0 \end{bmatrix},$$

$$\begin{bmatrix} \psi_{i,t} \\ \psi^*_{i,t} \end{bmatrix} = \rho \begin{bmatrix} \cos\lambda_c & \sin\lambda_c \\ -\sin\lambda_c & \cos\lambda_c \end{bmatrix} \begin{bmatrix} \psi_{i,t-1} \\ \psi^*_{i,t-1} \end{bmatrix} + \begin{bmatrix} \psi_{i-1,t} \\ 0 \end{bmatrix}, \tag{7}$$

$$i = 2, \ldots, n.$$

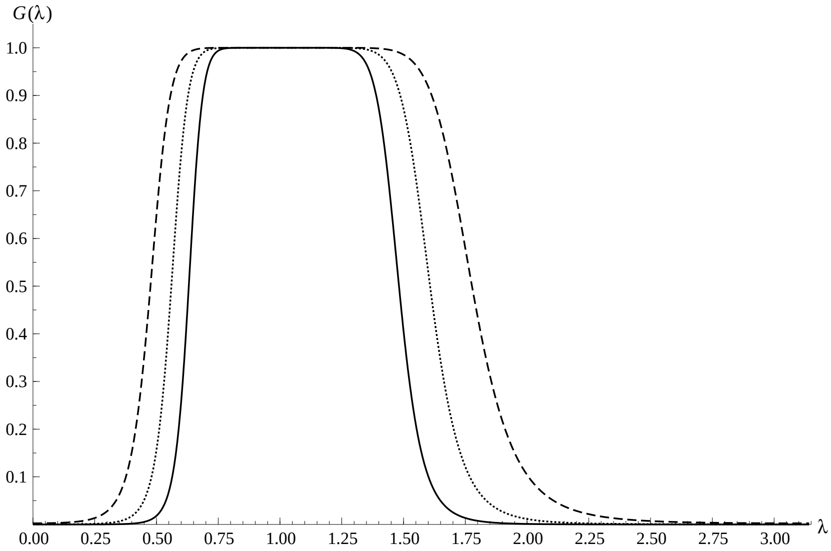

Figure 3: Gain of the Butterworth band-pass filter for $\lambda_c = 1$, $n = 6$ and $q = 0.1$ (solid), $q = 1$ (dotted) and $q = 10$ (dashed).

This process is stationary when the *dumping factor* ρ, defined on the interval $(0, 1]$, is strictly smaller than 1, and in what follows $\psi_{n,t}$ will be referred to as *n-order stochastic cycle*.

The gain function of the MSE-optimal filter to extract the component (7) when buried in white noise is

$$GB^{\mathrm{bp}}_{n,\rho,\lambda_c}(\lambda) = \frac{q_k[g_\psi(\lambda;\rho,\lambda_c)]^n}{1 + q_k[g_\psi(\lambda;\rho,\lambda_c)]^n} \tag{8}$$

where

$$g_\psi(\lambda;\rho,\lambda_c) = \frac{1 + \rho^2\cos^2\lambda_c - 2\rho\cos\lambda_c\cos\lambda}{1 + \rho^4 + 4\rho^2\cos^2\lambda_c - 4\rho(1+\rho^2)\cos\lambda_c\cos\lambda + 2\rho^2\cos 2\lambda}. \tag{9}$$

The effect of the dumping factor ρ on the gain is similar to joint effects of the pair q_κ e n: the smaller ρ, the less rectangular is the gain and the wider is the band-width.

2.2 Filters deriving from structural models

A structural model is an unobservable components model, for which the latent processes are specified a priori, on the basis of the features that the components are expected to show. The component processes are ARIMA processes with opportunely chosen constrains on the parameters. The structural model recently proposed by Harvey and Trimbur (2003) is

$$y_t = \mu_{m,t} + \psi_{n,t} + \varepsilon_t \qquad t = 1, \ldots, T, \tag{10}$$

where $\mu_{m,t}$, $\psi_{n,t}$ and ε_t are respectively an m-order stochastic trend, an n-order stochastic cycle and a white noise; the three processes are supposed to be not correlated. A seasonal component could be added to model (10) in order to avoid the distortions that the seasonal adjustment procedures may induce on the components.

When filters are not ideal, the signals extracted by them change if the order of extraction of each component vary, so it would be nice to have a filter that extracts the different components simultaneously. The MSE-optimal filter to simultaneously extract the components of the process (10) obtained by Harvey and Trimbur (2003) is therein named *Generalized Butterworth filter of order* (m, n). The gain function of the low-pass part is

$$GB^{lp}_{m,n}(\lambda) = \frac{q_\zeta(2 - 2\cos\lambda)^{-m}}{q_\zeta(2 - 2\cos\lambda)^{-m} + q_\kappa[g_\psi(\lambda; \rho, \lambda_c)]^n + 1}, \tag{11}$$

while the gain of the band-pass filter is

$$GB^{bp}_{m,n}(\lambda) = \frac{q_\kappa[g_\psi(\lambda; \rho, \lambda_c)]^n}{q_\zeta(2 - 2\cos\lambda)^{-m} + q_\kappa[g_\psi(\lambda; \rho, \lambda_c)]^n + 1}; \tag{12}$$

the two functions can be seen in figure 4.

The model described in equations (10), (5) and (7) can be easily stated in state-space form, its component extracted by the Kalman smoother and the unknown parameters estimated through maximum likelihood.

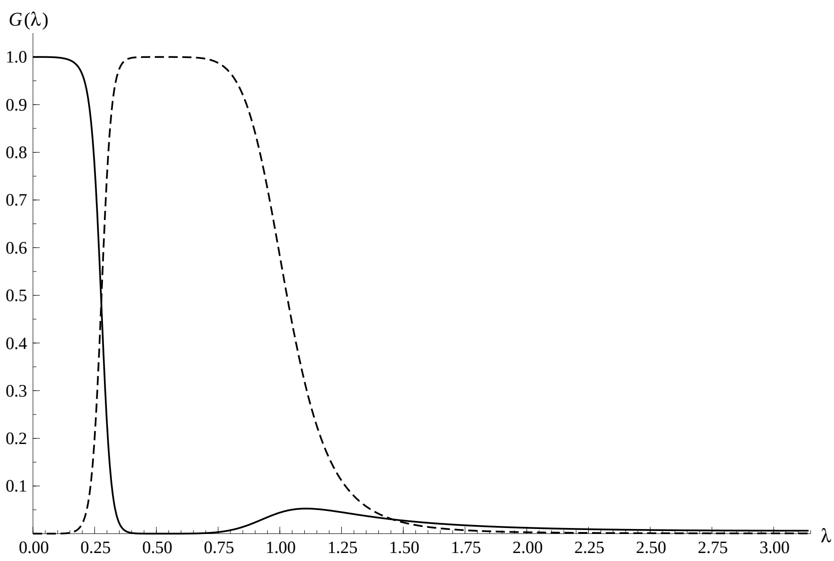

Figure 4: Generalized Butterworth filter: gains of the low-pass filter (solid) and of the band-pass filter (dashed) for $\lambda_c = 0.5$, $m = 2$, $n = 4$, $\rho = 0.9$ e $q_\kappa = q_\zeta = 0.1$.

3 A method for the extraction of the business cycle and the sector cycles

As already mentioned, the (macro) business cycle is defined as a fluctuation with periods in the range 1.5-8 years, common to many economic sectors. This definition allows for the possibility that each variable affected by the business cycle may have movements at business cycle frequencies not common to other variables. In what follows, it will be referred to these idiosyncratic cycles as *sector cycles*.

In this section a methodology for extracting the business cycle from a pool of sector time series is proposed. Once the original time series have been "cleaned" from the common business cycle, filters like the ones illustrated in previous section are used to extract the remaining sector cycles. Extracting the business cycle and the sector cycles simultaneously is theoretically possible (Koopman and Valle e Azevedo 2004), but when the model is applied to more then two or three time series, the number of parameters to be estimated increases considerably, making the estimates unstable and unreliable.

Extraction of the common cycle

The multivariate model used to extract the common cycle from the sector time series is[2]

$$y_{k,t} = \mu_{k,t} + \delta_k \psi_t + \varepsilon_{k,t} \tag{13}$$

for $k = 1, \ldots, K$ sectors, where $\mu_{k,t}$ is the m_k-order stochastic trend of the k-th sector, ψ_t is an n-order stochastic cycle common to all sectors, weighted with unknown loadings δ_k, and $\varepsilon_{k,t}$ are uncorrelated white noises. In order to identify the model a loading parameter must be fixed: here $\delta_1 = 1$, so that the cycle will have the same unit of measurement as $y_{1,t}$.

Since the component ψ_t must capture only the common fluctuations at business cycle frequencies, it is not a bad idea to choose $m = 1$ as trend order; in fact the filter built for such a trend has a relatively high gain also at business cycle frequencies, and it captures variability at business cycle frequencies that is not common to all the time series (the rest of idiosyncratic variability is captured by the white noise component).

Extraction of the sector cycles

To extract the sector cycle of the k-th sector a Generalized Butterworth filter is applied to the respective time series cleaned of the common cycle:

$$\tilde{y}_{k,t} = y_{k,t} - \delta_k \psi_t. \tag{14}$$

Harvey and Trimbur (2003) suggest to estimate all the parameters of the unobservable components, but even in some applications in their article they get

[2]In order to lighten the notation, the footers indicating the orders of the trend and of the cycle will be neglected.

unreasonable values of λ_c. Since the main goal of this work is extracting cycle components with frequencies coherent with the business cycle definition, here the values of the parameters λ_c and ρ are priorly fixed to cover the desired band.

4 The Italian business cycle and sector cycles

The technique proposed in the previous section has been applied to nine quarterly time series of the Italian value added (constant market prices, seasonally adjusted) for the following sectors

1. food, beverages and tobacco industries (food),
2. textiles, clothing, leather and shoes (text),
3. chemical industries (chem),
4. metal and metal products (metl),
5. mechanical industries (mech),
6. vehicles (vehi),
7. commerce, lodging, public exercises (comm),
8. transport and communication services (tran),
9. credit, insurances, real estate, professional services (cred),

furthermore, the time series of total value added (VA) has been used with loading $\delta_{va} = 1$ so that the extracted common cycle unit of measurement has an immediate interpretation.

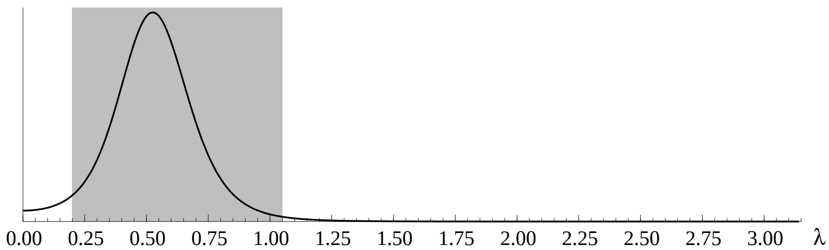

Figure 5: Spectrum of the cycle process used in the application. The gray shade emphasize the business cycle frequencies.

The business cycle frequencies for a quarterly time series are in the band [0.20,1.05]. The parameters fixed in the models are $m = 1$, $n = 4$, $\lambda_c = 0.5$ and $\rho = 0.7$. Since in the multivariate context it is not easy to define gain functions, figure 5 reports the power spectrum (Harvey and Trimbur 2003, equation 11) of the stochastic cycle used in the application.

In order to lighten the computational burden and avoid to get stuck in local maxima as much as possible, the estimation process has been incremental: first

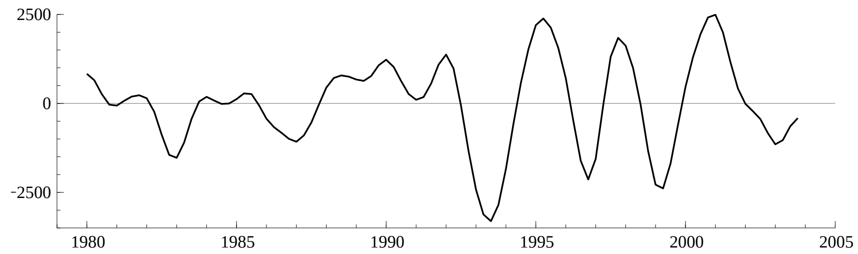

Figure 6: Final common cycle.

a model only with the total VA time series has been estimated, then one variable at time has been added using the estimates of the previous step as starting values. This procedure has led to final reasonable estimates without troubles. As expected, the common cycle component has shown increasing smoothness at every new step. The final common cycle is shown in figure 6.

It is interesting to analyze the role that the business cycle plays in each sector. The 3$^{\text{rd}}$ column of table 1 lists the cycle loadings δ_k, but since the scale factors in each sector are not homogeneous (evident from column 2), the last column reports the following scale-adjusted loadings

$$\tilde{\delta}_k = \delta_k \frac{\sigma(\Delta y_{va,t})}{\sigma(\Delta y_{\mathrm{k},t})},$$

where $\sigma(x_t)$ stands for the empirical standard deviation of the time series $\{x_t\}$ and Δ is the ordinary difference operator.

The sector that is at most influenced by the business cycle is commerce and public exercises, which (in the homogeneous scale) amplifies the business cycle signal. The sectors of mechanical industries, vehicles, clothing and metal industries are also strongly bound up with the business cycle, while transport & communication services, credit & insurances, chemical and food industries show a weaker relation with it (from a statistical point of view, the null hypothesis $\delta_k = 0$ could not be rejected at a 5% significance level).

The sector cycles and trends have been extracted with generalized Butterworth filters of order $(2, 4)$ and fixed parameters $\rho = 0.7$, $\lambda_c = 0.5$. These filters have been applied to the sector series, cleaned up from the common cycle as described in the previous section. Figures 7-11 report, on the top, each sector's trend with the original series net of the common cycle, on the bottom, the idiosyncratic cycle and the common cycle weighted with the respective loading.

From a structural point of view, the textile & clothing sector experiences a deep crisis due to many concurring reasons: the production moving into low labour-cost countries, a growing counterfiet industry, a strong Euro currency, and a hardened national and international competition narrowing the profit margins. Also the mechanical industry trend is dropping at an increasingly faster rate starting from year

sector k	$\sigma(\Delta y_{k,t})$	loading δ_k	loading $\tilde{\delta}_k$
comm	249	0.24	1.32
va	*1350*	*1.00*	*1.00*
mech	210	0.15	0.98
vehi	86	0.06	0.95
text	137	0.09	0.86
metl	194	0.10	0.71
tran	265	0.07	0.35
chem	99	0.02	0.29
cred	314	0.06	0.25
food	139	0.01	0.12

Table 1: Standard deviations of first differences (2nd column), loadings (3rd column) and scale-adjusted loadings (4th column).

2001. Chemical industry, vehicles and commerce show a flat trend at the end of the period. The most healthy sectors are the food and beverage industry, the metal industry, credit & insurance services and, with a weaker growth, transport & communications.

From a conjunctural point of view, there are sectors such as the mechanic and metal industries, and transport and communications, for which the only relevant cycle is the common cycle. The food and beverages industry and the chemical industry have idiosyncratic cycles that "explain" more variance than the common cycle. In these sectors a general counter-cyclical policy could result not so effective and *ad hoc* policies should be undertaken. The remaining sectors have significant idiosyncratic cycles, although not so important as the common cycle.

The last two graphs of figure 11 have been included to show that the common cycle filter has effectively extracted the whole business cycle: in fact, the total VA time series, cleaned of the common cycle, does not show significative variance at business cycle frequencies.

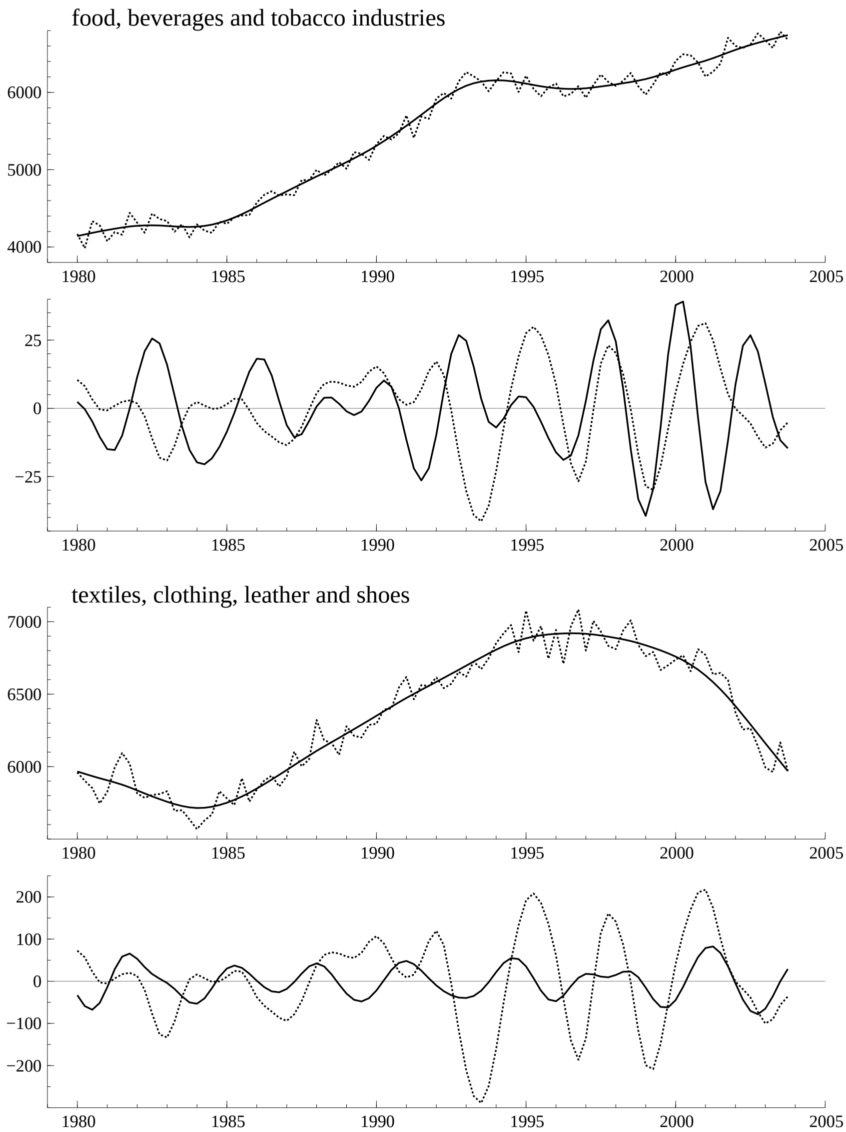

Figure 7: Graphs in first and third position: sector trend (solid) and VA series cleaned of the common cycle (dashed). Graphs in second and fourth position: sector cycle (solid) and common cycle multiplied times the respective loading (dashed).

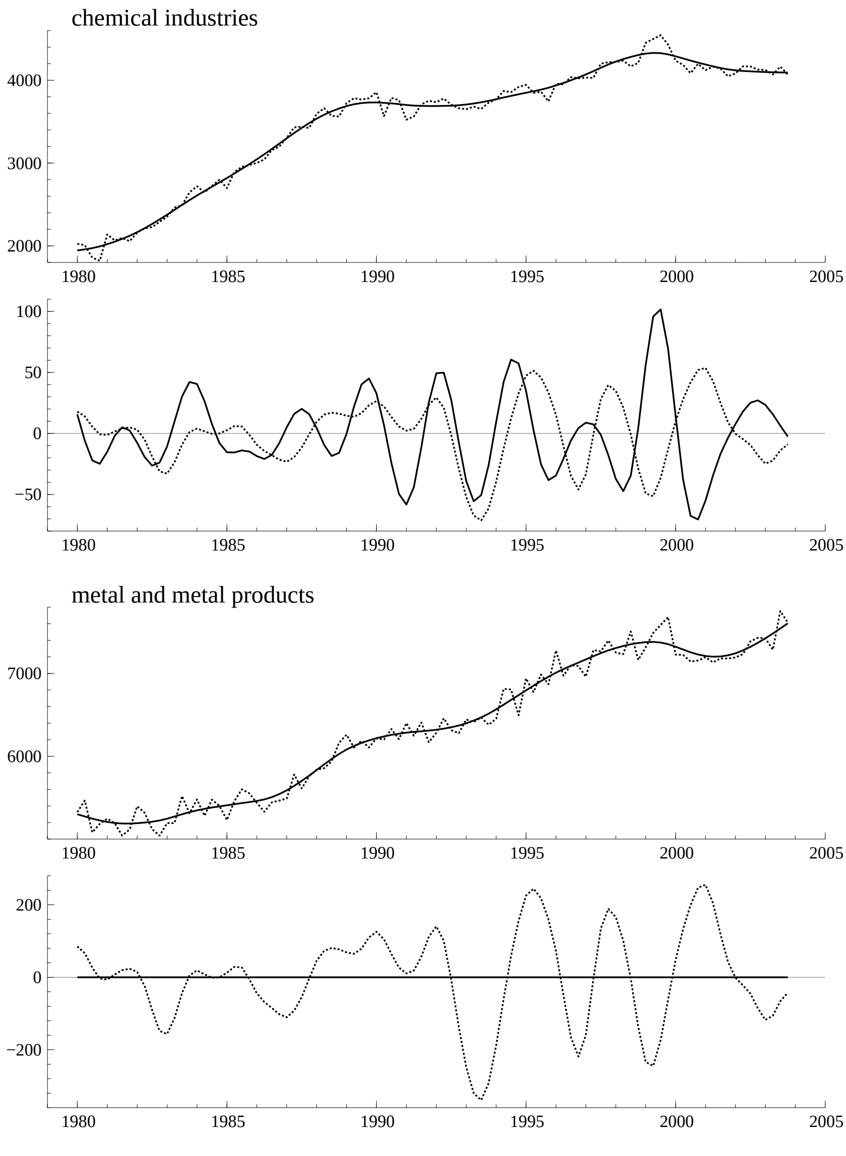

Figure 8: Graphs in first and third position: sector trend (solid) and VA series cleaned of the common cycle (dashed). Graphs in second and fourth position: sector cycle (solid) and common cycle multiplied times the respective loading (dashed).

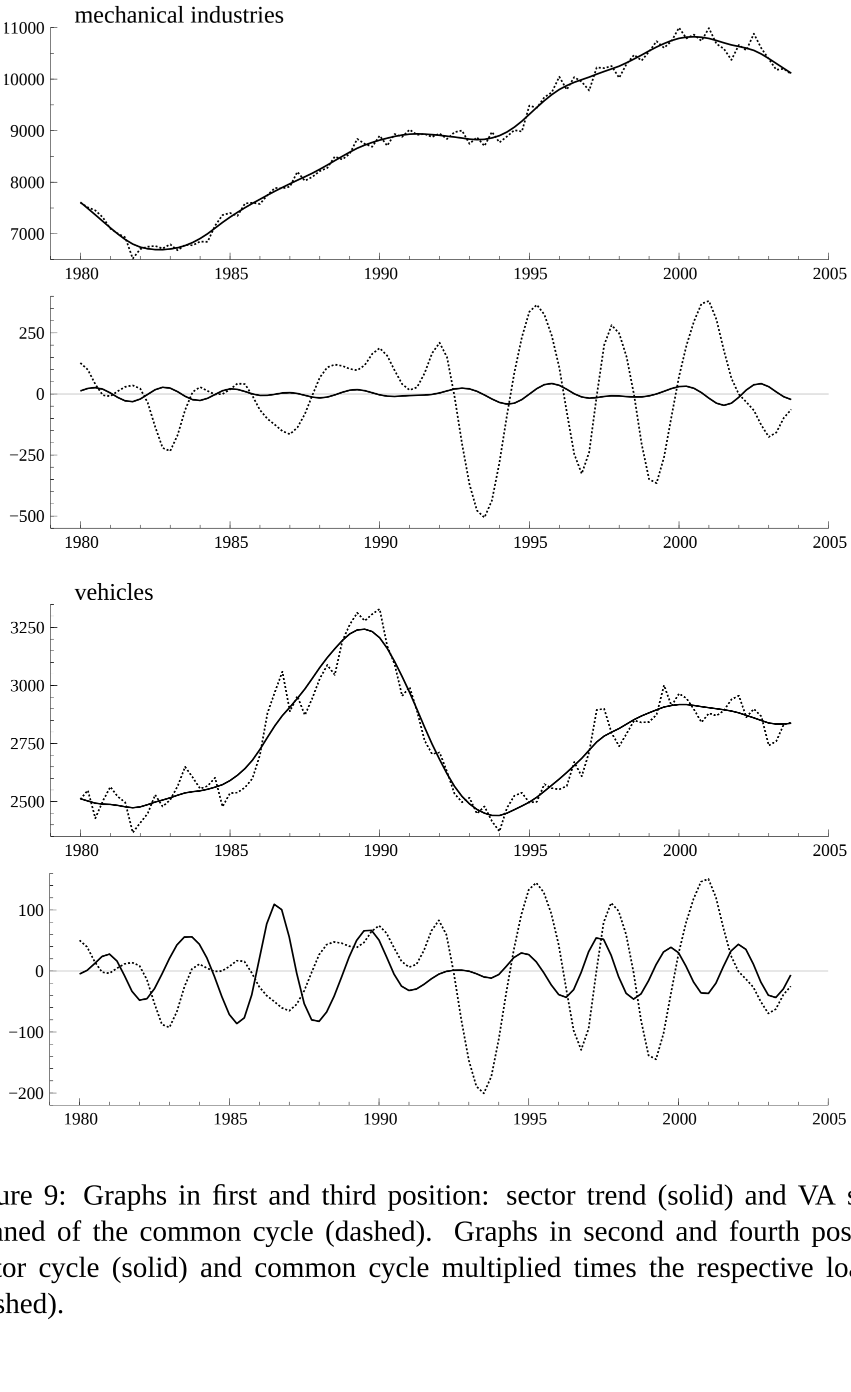

Figure 9: Graphs in first and third position: sector trend (solid) and VA series cleaned of the common cycle (dashed). Graphs in second and fourth position: sector cycle (solid) and common cycle multiplied times the respective loading (dashed).

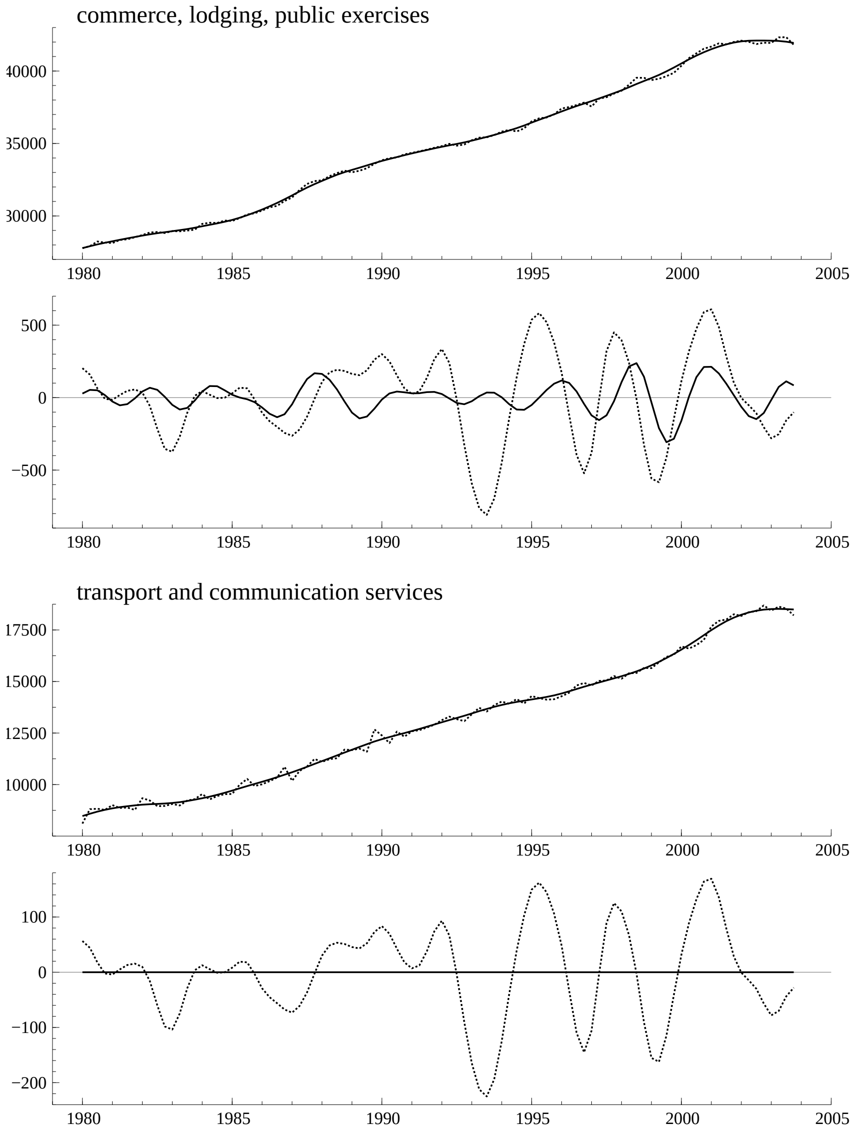

Figure 10: Graphs in first and third position: sector trend (solid) and VA series cleaned of the common cycle (dashed). Graphs in second and fourth position: sector cycle (solid) and common cycle multiplied times the respective loading (dashed).

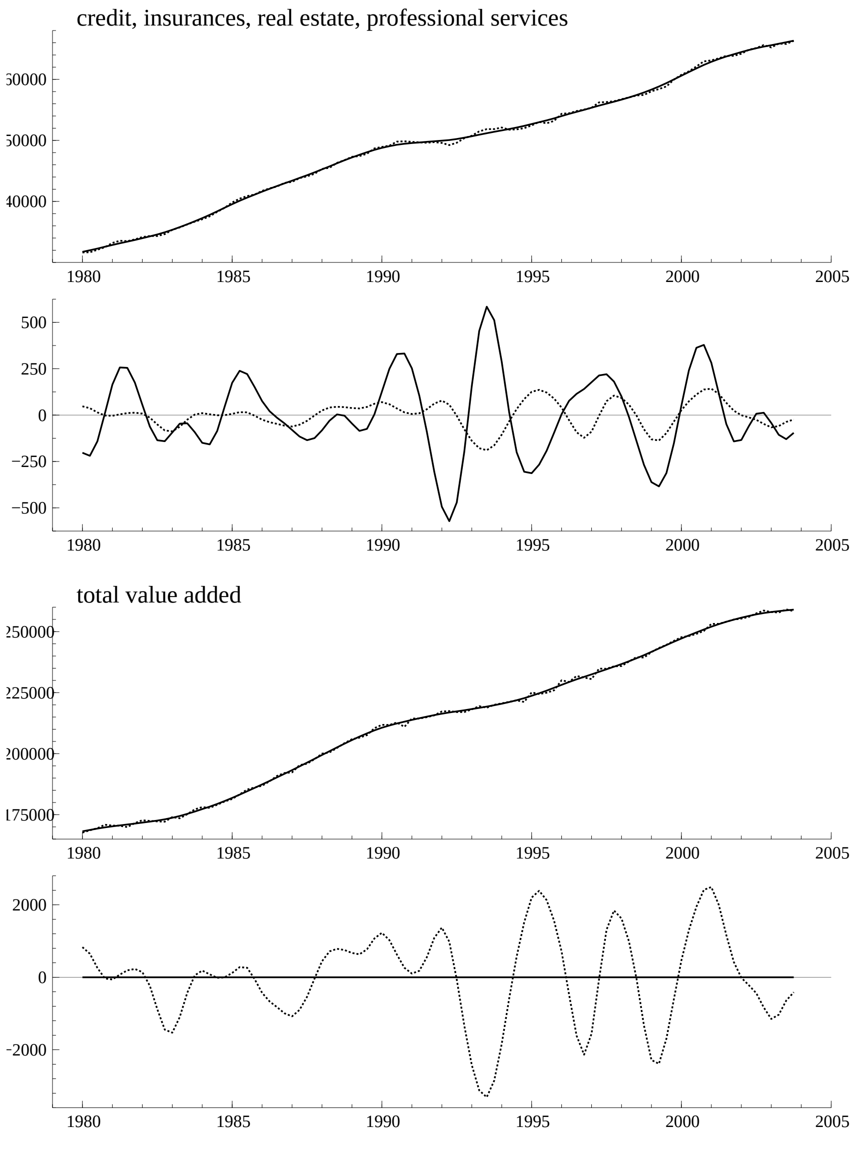

Figure 11: Graphs in first and third position: sector trend (solid) and VA series cleaned of the common cycle (dashed). Graphs in second and fourth position: sector cycle (solid) and common cycle multiplied times the respective loading (dashed).

Appendix: state space representation of the model

The state space representation of the generalized Butterworth filter can be found in Harvey and Trimbur (2003). Here it is shown how to cast the multivariate model (13) in state space form.

Let n be the stochastic cycle order, m the stochastic trend order, K the number of time series in the model, $\boldsymbol{U}_s$ a squared upper triangular matrix of ones of dimension $s \times s$, and J_s a squared matrix of ones of dimension $s \times s$.

Let

$$\boldsymbol{\psi}_t = \begin{bmatrix} \psi_{n,t} \\ \psi^*_{n,t} \\ \vdots \\ \psi_{1,t} \\ \psi^*_{1,t} \end{bmatrix}, \quad \boldsymbol{\mu}^{(k)}_t = \begin{bmatrix} \mu^{(k)}_{m_k,t} \\ \vdots \\ \mu^{(k)}_{1,t} \end{bmatrix}, \quad \boldsymbol{\mu}_t = \begin{bmatrix} \boldsymbol{\mu}^{(1)}_t \\ \vdots \\ \boldsymbol{\mu}^{(K)}_t \end{bmatrix},$$

be, respectively, the state-vector of the common cycle, the state-vector of the k-th sector's trend for $k = 1, \ldots, K$, and the state-vector containing all the K trends.

The state-vector for the whole model is

$$\boldsymbol{\alpha}_t = \begin{bmatrix} \boldsymbol{\psi}_t \\ \boldsymbol{\mu}_t \end{bmatrix}.$$

The transition matrix for the stochastic cycle is

$$\boldsymbol{T}_\psi = \boldsymbol{U}_n \otimes \begin{bmatrix} \rho\cos\lambda_c & \rho\sin\lambda_c \\ 0 & 0 \end{bmatrix} + \boldsymbol{I}_n \otimes \begin{bmatrix} 0 & 0 \\ -\rho\sin\lambda_c & \rho\cos\lambda_c \end{bmatrix},$$

and the covariance matrix of the common cycle's disturbances is

$$\boldsymbol{\Sigma}_\psi = \boldsymbol{J}_n \otimes \begin{bmatrix} \sigma^2_k & 0 \\ 0 & 0 \end{bmatrix}.$$

The transition matrix and the covariance matrix of the k-th trend are, respectively

$$\boldsymbol{T}_{\mu_k} = \boldsymbol{U}_m, \qquad \boldsymbol{\Sigma}_{\mu_k} = \boldsymbol{J}_m \sigma^2_{\zeta_k}.$$

The transition matrix and the covariance matrix of the whole state vector are

$$\boldsymbol{T} = \begin{bmatrix} \boldsymbol{T}_\psi & \mathbf{0} & \ldots & \mathbf{0} \\ \mathbf{0} & \boldsymbol{T}_{\mu_1} \ldots & \mathbf{0} & \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \ldots & \boldsymbol{T}_{\mu_K} \end{bmatrix}, \quad \boldsymbol{\Sigma}_\alpha = \begin{bmatrix} \boldsymbol{\Sigma}_\psi & \mathbf{0} & \ldots & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Sigma}_{\mu_1} \ldots & \mathbf{0} & \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \ldots & \boldsymbol{\Sigma}_{\mu_K} \end{bmatrix}.$$

The system matrix of the observation equations is

$$\boldsymbol{Z} = [\boldsymbol{z} \quad \mathbf{0}_{K\times(2n-1)} \quad \boldsymbol{e}_1 \quad \mathbf{0}_{K\times(m-1)} \quad \boldsymbol{e}_2 \quad \mathbf{0}_{K\times(m-1)} \quad \ldots \quad \boldsymbol{e}_K \quad \mathbf{0}_{K\times(m-1)}]\ ,$$

where $\boldsymbol{z} = [1 \;\; z_2 \;\; \ldots \;\; z_K]'$ is a vector of loadings, $\boldsymbol{e}_i$ is the i-th column of the identity matrix I_K, and $\mathbf{0}_{r \times s}$ is an $r \times s$ matrix of zeros. The covariance matrix of the measurement errors, $\boldsymbol{\Sigma}_\varepsilon$, is diagonal.

The Kalman filter can be initialized with a zero mean-vector and a covariance matrix

$$\begin{bmatrix} \boldsymbol{\Gamma} & \mathbf{0} \\ \mathbf{0} & \gamma \boldsymbol{I}_{m \cdot K} \end{bmatrix},$$

where $\boldsymbol{\Gamma}$ is the unconditional covariance matrix of the process $\boldsymbol{\psi}_t$, given by

$$\mathrm{vec}\, \boldsymbol{\Gamma} = [\boldsymbol{I}_{4n^2} - \boldsymbol{T}_\psi \otimes \boldsymbol{T}_\psi]^{-1} \mathrm{vec}\, \boldsymbol{\Sigma}_\psi,$$

and $\gamma \to \infty$, giving the non-stationary components a diffuse initial distribution.

The model has been implemented as an object-class for Ox (Doornik 2001), based on the state space algorithms of the SsfPack of Koopman et al. (1999), available from the author on request.

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