Università Carlo Cattaneo-Castellanza (LIUC)
Faculty of Economics
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Quantitative Methods for Economics, Finance and Management

(Instructors: Prof. Marzio Galeotti, Prof. Matteo Manera)

Take-home 2

(December 4, 2009)

Instructions for Take-home 1

Answer to all questions listed below. Hand-in your answers as an attachment of a e-mail message to Prof. Marzio Galeotti (e-mail: marzio.galeotti@unimi.it) and to Prof. Matteo Manera (e-mail: matteo.manera@gmail.com) by December 7, 2009, h. 11pm.

Question 1

You are given the linear regression model:

(1) $Y_t = \beta Y_{t-1} + u_t$

(2) $u_t = \rho u_{t-1} + \varepsilon_t$

where t=2,...,T; ε_t are serially independent error terms; -1<β<1; -1<ρ<1.

1.1) Show that

$$E(u_t Y_{t-1}) = \frac{\rho \sigma_u^2}{1 - \rho\beta}$$

1.2) What are the consequences of this result for the OLS estimate of β?

1.3) Let $\hat{\beta}$ be the OLS estimator of β in equation (1). Show that

$$plim(\hat{\beta}) = \frac{\beta + \rho}{1 + \beta\rho}$$

Question 2

2.1) Consider the following regression equation without a constant term:

$$Y_i = \beta X_i + u_i$$

with i=1,...,N. Under what assumptions on the variance of the errors u_i are expressions (3) and (4) the best linear unbiased estimators of β?

(3) $\frac{1}{N}\sum_{i=1}^{N}\frac{Y_i}{X_i}$

(4) $\frac{\sum_{i=1}^{N} Y_i}{\sum_{i=1}^{N} X_i}$

2.2) Assume that the economic model of interest is a micro-relationship of the type:

(5) $Y_{ij} = \beta_1 + \beta_2 X_{ij} + u_{ij}$

where the subscripts refer to the ith firm in the jth industry. Unfortunately, it is not possible to estimate model (5), since data at firms' level are unavailable. The only variables which are observed are at industry level (i.e. aggregated data), that is the only estimable model is:

(6) $Y_{.j} = \beta_1 + \beta_2 X_{.j} + u_{.j}$

where, in general, $z_{.j} = \frac{1}{n_j}\sum_{i=1}^{n_j} z_{ij}$, and n_j indicates the number of firms in the jth industry.

2.2.1) Show that, if $var(u_{ij}) = \sigma^2$ (constant), then $var(u_{.j}) = \sigma^2/n_j$. What is the implication of this result?

2.2.2) What is the transformation on model (6) which guarantees that the OLS estimator on the transformed model (6) is BLUE?

Question 3

A researcher is given 20 observations on consumption (Y), income (Z) and investment (X). The constant term is indicated with C. He/she estimates with OLS the following regression:

Table 1
Dependent Variable: Y
Method: Least Squares

Sample: 1 20
Included observations: 20

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	1.471809	0.463538	3.175166	0.0052
Z	0.822890	0.018673	44.06884	0.0000

R-squared	0.990817	Mean dependent var	21.74700
Adjusted R-squared	0.990306	S.D. dependent var	2.566870
S.E. of regression	0.252724	Akaike info criterion	0.181599
Sum squared resid	1.149646	Schwarz criterion	0.281172
Log likelihood	0.184015	F-statistic	1942.062
Durbin-Watson stat	1.453617	Prob(F-statistic)	0.000000

3.1) With reference to the results reported in Table 1, which variables are statistically significant and why?

3.2) What is the interpretation of F-statistic and of Prob(F-statistic) reported in Table 1?

3.3) What are the implications on the OLS estimates presented in Table 1 of the assumption that the variable Z is contemporaneously correlated with the errors of the regression of Y on C and Z?

3.4) The researcher thinks that variable X is an appropriate instrument for Z. What does this mean?

3.5) The researcher uses the following alternative procedure to estimate the parameters presented in Table 1:

Table 2
Dependent Variable: Z
Method: Least Squares

Sample: 1 20
Included observations: 20

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	10.52918	1.407752	7.479429	0.0000
X	4.865455	0.476179	10.21770	0.0000

R-squared	0.852943	Mean dependent var	24.63900
Adjusted R-squared	0.844773	S.D. dependent var	3.104979
S.E. of regression	1.223327	Akaike info criterion	3.335664
Sum squared resid	26.93750	Schwarz criterion	3.435238
Log likelihood	-31.35664	F-statistic	104.4013
Durbin-Watson stat	1.821979	Prob(F-statistic)	0.000000

Table 3

Dependent Variable: Y
Method: Least Squares

Sample: 1 20
Included observations: 20

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	2.273351	2.456086	0.925599	0.3669
ZHAT	0.790359	0.099048	7.979583	0.0000

R-squared	0.779611	Mean dependent var	21.74700
Adjusted R-squared	0.767367	S.D. dependent var	2.566870
S.E. of regression	1.238054	Akaike info criterion	3.359598
Sum squared resid	27.58998	Schwarz criterion	3.459171
Log likelihood	-31.59598	F-statistic	63.67374
Durbin-Watson stat	1.795330	Prob(F-statistic)	0.000000

3.6) Given that ZHAT are the fitted values from the regression presented in Table 2, explain the motivation of running the two regressions reported in Tables 2 and 3.

3.7) Explain the interpretation of testing the null hypothesis that the coefficient of ZHAT is zero in the regression model reported in Table 4. Is this null hypothesis rejected?

Table 4

Dependent Variable: Y
Method: Least Squares

Sample: 1 20
Included observations: 20

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	2.273351	0.076487	29.72187	0.0000
Z	1.011575	0.007429	136.1729	0.0000
ZHAT	-0.221216	0.008044	-27.50232	0.0000

R-squared	0.999798	Mean dependent var	21.74700
Adjusted R-squared	0.999774	S.D. dependent var	2.566870
S.E. of regression	0.038555	Akaike info criterion	-3.535956
Sum squared resid	0.025271	Schwarz criterion	-3.386596
Log likelihood	38.35956	F-statistic	42098.96
Durbin-Watson stat	2.735371	Prob(F-statistic)	0.000000