THE FOURIER FORMULA FOR DISCONTINUOUS
FUNCTIONS OF SEVERAL VARIABLES

A.N. Podkorytov and Mai Van Minh

We would like to talk about following problem. Suppose Ω is convex compact
set in \( \mathbb{R}^m \) and \( \chi_\Omega \) its indicator. Obviously its Fourier transformation \( \hat{\chi}_\Omega \) is not
summable in \( \mathbb{R}^m \), if \( \text{Int}(\Omega) \neq \emptyset \). The problem is to interpret the integral in the
R.H.S. of the inversion formula

\[
\chi_\Omega(y_0) = \int_{\mathbb{R}^m} \hat{\chi}_\Omega(x)e^{-2\pi i x \cdot y_0} dx,
\]

while keeping the equality true. For example if \( \Omega = \square = [a_1, b_1] \times \ldots \times [a_m, b_m] \) is
a rectangular parallelepiped it is not hard to verify that

\[
\int_{[-R,R]^m} \hat{\chi}_\square(x)e^{-2\pi i x \cdot y_0} dx \underset{R \to +\infty}{\longrightarrow} \chi_\square(y_0) \quad \text{for } y_0 \notin \partial \square.
\]

But as mentioned in [1], if \( \Omega = \bigcirc = \{ y \in \mathbb{R}^m | \| y \| \leq 1 \} \) is a sphere then the
situation becomes more complicated. If \( y_0 \neq 0 \) then

\[
\int_{\| x \| \leq R} \hat{\chi}_\bigcirc(x)e^{-2\pi i x \cdot y_0} dx \underset{R \to +\infty}{\longrightarrow} \chi_\bigcirc(y) \quad \text{for } y_0 \notin \partial \bigcirc
\]

(in case \( \| y_0 \| = 1 \) the limit equals \( \frac{1}{2} \)). At the same time when \( m \geq 3 \) at \( y_0 = 0 \) these
integrals do not have a limit. In [2] the sphere \( \{ \| x \| \leq R \} \) have been replaced by a cube and it has been shown that when \( m = 3 \) and \( y_0 = 0 \) the following equality is
correct \( 1 = \lim_{R \to +\infty} \int_{[-R,R]^3} \hat{\chi}_\bigcirc(x) dx \). In order to extend this result, we could prove
that for every convex compact set \( \Omega \subset \mathbb{R}^m, m \geq 2 \), the following equality is correct

\[
\chi_\Omega(y_0) = \lim_{R \to +\infty} \int_{x \in RW} \hat{\chi}_\Omega(x)e^{-2\pi i x \cdot y_0} dx \quad \text{for } y_0 \notin \partial \Omega,
\]

if \( W \) is polyhedron in \( \mathbb{R}^m \), \( 0 \in \text{Int}(W) \) and the sides of \( W \) and their extensions do
not cross the origin. If \( m = 2 \) we can take as \( W \) any compact convex neighborhood
of the origin instead of polygon. In this case the inversion formula remains the
same for \( y_0 \notin \partial \Omega \). And, what is more interesting, if \( W \) is simmetrical (relatively
the origin) at every point of \( y_0 \in \partial \Omega \) there is finite limit of integrals on \( RW \). It
equals \( \frac{1}{2} \), if \( y_0 \) is not boundary angular point (vertex). For boundary angular point
this limits depends on choice of \( W \).

[1] Pinsky M. A., Stanton N. K., Trapa P. E. Fourier series of radial functions
